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DATE ON BUCKLING STRENGTH OF CURVED SHEET IN COMPRESSION

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ADVANCE RESTRICTED REPORT

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By Harold Crate and L. Ross Levin

SUMMARY

Tests were made of curved panels of four different thicknesses and with radius-thickness ratios varying from about 150 to ∞. Results are also included of some tests that were reported previously.

The data presented lead to the conclusion that for practical engineering use the critical compressive stress for a curved sheet between stiffeners is given by the larger of the following:

(a) The critical compressive stress for an unstiffened circular cylinder of the same radius-thickness ratio

(b) The critical compressive stress for the same sheet when flat

It is indicated from the tests made that there is a certain value of radius-thickness ratio r/t, varying with the skin thickness, below which the critical stress for repeated loading is less than that obtained upon first application of the load. If, therefore, the value of r/t for a particular structural part is below this value, the part should be so designed that it will not buckle.

INTRODUCTION

Because the skin between stiffeners on the surface of airplanes is curved to the contour of the wing or fuselage, it is important to investigate the extent to which this curvature influences the critical stress.

Reference 1 presents a theoretical formula for the critical compressive stress of a slightly curved sheet with equal elastic restraints against rotation along the

straight, unloaded edges, and a semirational formula for cases in which the curvature is larger. The present report gives experimental data on the critical compressive stress of curved sheet and presents a modified formula based on the theoretical formula of reference 1.

TEST SPECIMENS

Each test specimen was constructed of a curved sheet of 24S-T aluminum alloy with four angle stiffeners, formed from flat sheet of the same material and attached along the straight, unloaded edges, as shown in figure 1. As all specimens were loaded within the elastic range of the material, the modulus of elasticity E is the only material property of concern. The value of E was assumed to be 10,600,000 pounds per square inch in all the calculations of this paper. The dimensions of the specimens are given in table I. The symbols used for the dimensions are those shown in figure 1. (In the radius-thickness ratio r/t used in table I, r is the radius of curvature of the sheet and t is the thickness of the sheet, designated t_s in fig. 1.)

The angle type of stiffener was selected because of the low rotational restraint that it provided at the side edges of the sheet. The use of two stiffeners at each side edge of the sheet was decided upon in order to stabilize thoroughly these edges against displacements normal to the sheet. The various sizes of angle were so selected as to force buckling to occur in the sheet at a load lower than the lowest critical load of the stiffeners and to provide adequate support against deflection normal to the sheet, without having excessive area in the stiffeners. In some cases, the proportions of the stiffeners in a particular series of test panels were changed after the test program was under way, in an effort to realize more closely the conditions just set forth. The test results indicated that such changes as were made had little effect on the experimental values of critical stress obtained.

The areas listed in table I, which were used for calculating average stress, were determined from the weights of the specimens and the density of the material.

TEST PROCEDURE

Specimens were tested in the 1,200,000-pound-capacity testing machine in the NACA structures research laboratory. In order to insure uniform bearing against the heads of the testing machine, the specimens were ground to produce ends which were flat, square, and parallel. After the specimen had been placed in the testing machine and a small initial load applied, the radius of curvature was measured with the gage shown in figure 2.

For all specimens except those of group D (see table I), strains on the two sides of the specimen were measured by means of eight pair of Tuckerman optical strain gages located as shown in figure 1. The specimens of group D are those used in the tests reported in reference 2; the strain-gage locations used are also given in reference 2.

In some cases after the sheet buckled, the load was released and then applied again, in order to determine whether or not the critical stress was the same under repeated loading as under a load applied only once. Sometimes this procedure was used in a single series of loads, and sometimes the panel was removed from the testing machine, the ends reground, and the entire test repeated.

METHOD OF DETERMINING CRITICAL BUCKLING LOAD

The method of determining the critical buckling load depended in each case on the action of the specimen. In some cases (specimens with low r/t) buckling occurred suddenly by a snap-diaphragm action accompanied by a loud report; the buckling load was then taken as the load at which this action occurred. In all other cases a gradual growth of deflections with load made a direct determination of the critical load impossible. Two methods were used to determine the critical load for these specimens.

The first method, explained in reference 3, is in brief a procedure for analyzing the gradual growth of deflections with load and consists in plotting a curve of $(y-y_1)/(P-P_1)$ as ordinate against $(y-y_1)$ as abscissa,

where P and y are corresponding values of load and deflection and P_1 and y_1 are arbitrarily chosen initial values of each quantity. The inverse slope of the straight line obtained is $P_{cr}-P_1$, where P_{cr} is the desired critical buckling load. The value of y was taken as the difference in readings of a pair of opposite strain gages located at or near the crest of the primary buckle.

The foregoing method gives a theoretical value of critical load for the specimen if it is perfect. A second method was employed that gives a practical value of critical load for the specimen with whatever imperfections it has. This method consists of plotting the difference of strain for a pair of strain gages near the crest of the buckle and visually estimating the critical stress as about the top of the knee of this curve, or the point at which a small increase of stress causes a relatively large increase in deflection. A typical curve, with the critical stress estimated by this method, is shown in figure 3. The critical stress determined by the first, or straight-line, method is also shown in figure 3 for purposes of comparison.

Both of these methods are based on the difference in strain-gage readings, which is a measure of the change of curvature. According to the theory of small deflections, which applies for stresses below the buckling load, the deflection of a given buckled shape is proportional to the curvature. It was considered that the strain readings of the Tuckerman optical strain gages were more accurate than the deflection readings that could be obtained with the equipment at hand.

DISCUSSION OF RESULTS

The results of these tests are given in table II and in figure 4, where σ_{cr} is the critical stress. The various points plotted in these figures show the critical stresses as determined by the three methods previously mentioned and the values of critical stress under repeated loading where such values were observed.

The curves A, B, and C in figure 4 are the curves given in figures 7 and 9 of reference 4, which represents the NACA study of circular cylinders in compression. Curves A and B, respectively, are the graphs of the equations

$$\frac{\sigma_{cr}}{E} = 0.605\ \frac{t}{r}$$

and

$$\frac{\sigma_{cr}}{E} = 0.363\ \frac{t}{r}$$

The plotted points in figures 7 and 9 of reference 4, representing the compressive strength of carefully constructed cylinders, scattered between curves B and C.

Curve D is a graph of equation (13) of reference 1. Curve E is a graph of the equation

$$\frac{\sigma_{cr}}{E} = 0.6\left[\frac{k'\pi^2}{12(1-\mu^2)}\left(\frac{t}{b}\right)^2 + \frac{1}{k'\pi^2}\left(\frac{b}{r}\right)^2\right] \tag{1}$$

where

$k' = \dfrac{k}{0.6}$

k coefficient in formula for critical stress of sheet when flat, $\dfrac{\sigma_{cr}}{E} = \dfrac{k\pi^2}{12(1-\mu^2)}\left(\dfrac{t}{b}\right)^2$

μ Poisson's ratio for material

b width of sheet between outstanding flanges of stiffeners

Equation (1) is a modification of equation (10) of reference 1, which is a generalization of equation (276) of reference 5 to include all degrees of edge restraint instead of simple support alone. The value of k was chosen to make the curve agree with the experimental points obtained by the straight-line method for $r/t = \infty$. The reason that curves D and E do not always pass through these points exactly is that the value of t_s for the particular specimens with $r/t = \infty$ was used in establishing k; whereas an average value of t_s was used in plotting the curves.

Regardless of which method is used to establish experimental critical stresses, it appears from the data that the effect of curvature cannot be relied upon to follow consistently the gradual increase in critical stress with increase in curvature represented by either of the curves D and E. Perhaps a more practical value for the critical compressive stress for a curved sheet between stiffeners would be given by the larger of the following:

(a) The critical compressive stress for an unstiffened circular cylinder of the same radius-thickness ratio

(b) The critical compressive stress for the same sheet when flat

The critical stress for the second and subsequent loadings was affected by the extent to which the buckles were made permanent under the first application of load.

For small values of r/t, buckling usually occurred with a snap-diaphragm action, indicating a drop in load when buckling occurred. The large deflection associated with the snap-diaphragm action produced relatively large permanent deformations in the specimen and, as a consequence, the critical stress for the second and subsequent loadings was appreciably less than for the first loading.

For large values of r/t where the deflection increased gradually with load, the first loading had no appreciable effect on the critical stress for further loading. If, however, the loading had been continued after buckling until permanent deformations had been produced in the specimen, the critical stress for subsequent loadings probably would have been less.

The critical stresses for repeated loads given in this report are consequently of qualitative rather than quantitative values for use in design, because no measurements of the deformations following buckling were made.

CONCLUSIONS

The data presented in this report lead to the conclusion that for practical engineering use the critical compressive stress for a curved sheet between stiffeners is given by the larger of the following:

1. The critical compressive stress for an unstiffened circular cylinder of the same radius-thickness ratio

2. The critical compressive stress for the same sheet when flat

The critical stress for the second and subsequent loadings may be affected by the deformation that occurs when the sheet buckles under the first application of load.

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TABLE I. - DIMENSIONS OF TEST SPECIMENS

Specimen	t_s (in.)	r (in.)	r/t	A (in.)	B (in.)	t_a (in.)	Area (sq in.)
Group A - Nominal t_s = 0.125 in.							
A-1	0.1255	∞	∞	0.99	0.81	0.1016	1.771
A-2	.1226	∞	∞	.99	.81	.1012	1.742
A-3	.1274	106.9	839	.99	.81	.0996	1.798
A-4	.1256	97.4	776	.98	.80	.1007	1.778
A-5	.1251	90.0	719	.99	.81	.1006	1.776
A-6	.1268	88.0	694	.98	.82	.1008	1.793
A-7	.1238	76.0	614	.99	.82	.1001	1.759
A-8	.1270	66.6	524	.99	.81	.1003	1.788
A-9	.1268	57.0	450	.98	.81	.0982	1.778
A-10	.1285	45.8	356	.98	.82	.1001	1.801
A-11	.1268	34.0	268	.99	.81	.1000	1.786
A-12	.1276	27.8	218	1.00	.81	.1007	1.804
A-13	.1270	16.8	132	.99	.81	.1005	1.788
Group B - Nominal t_s = 0.102 in.							
B-1	0.1031	∞	∞	0.99	0.75	0.0895	1.497
B-2	.1004	∞	∞	.99	.75	.0913	1.485
B-3	.0990	121.5	1227	.99	.76	.0917	1.473
B-4	.1015	97.1	957	.99	.75	.0917	1.493
B-5	.1029	96.2	935	.98	.75	.0926	1.513
B-6	.0993	91.5	922	.99	.75	.0921	1.483
B-7	.1014	77.1	760	.99	.75	.0898	1.480
B-8	.1003	73.4	732	.98	.75	.0923	1.489
B-9	.1030	75.1	729	.99	.75	.0882	1.494
B-10	.1007	63.4	630	.99	.75	.0922	1.495
B-11	.0993	54.7	551	.99	.76	.0906	1.470
B-12	.1015	37.5	370	1.01	.76	.0909	1.486
B-13	.1017	26.2	258	.99	.75	.0905	1.490
B-14	.0996	16.2	163	.99	.75	.0905	1.466
Group C - Nominal t_s = 0.081 in.							
C-1	0.0821	∞	∞	1.25	0.63	0.0640	1.223
C-2	.0818	98.7	1206	1.25	.62	.0640	1.223
C-3	.0813	59.9	737	1.26	.59	.0635	1.220
C-4	.0815	46.3	568	1.23	.64	.0643	1.228
C-5	.0816	42.0	513	1.24	.63	.0637	1.224
C-6	.0808	34.5	427	1.24	.64	.0632	1.214
C-7	.0814	32.7	402	.95	.60	.0640	1.145
C-8	.0810	28.2	348	.96	.60	.0641	1.137
C-9	.0807	24.2	300	.99	.76	.0902	1.317
C-10	.0810	22.1	273	.95	.60	.0636	1.142
C-11	.0807	17.1	212	.96	.60	.0636	1.138
C-12	.0810	15.2	188	.99	.75	.0893	1.291
C-13	.0817	11.7	143	.99	.75	.0902	1.309
Group D - Nominal t_s = 0.078 in.[a]							
D-1	[b].078	∞	∞	1.51	0.61	[b]0.078	1.310
D-2		103.0	1318	1.51	.61		1.310
D-3		63.3	806	1.08	.61		1.158
D-4		49.5	634	1.08	.61		1.155
D-5		42.2	542	.87	.61		1.103
D-6		37.3	478	.87	.61		1.102
D-7		33.8	432	.75	.61		1.067
D-8		31.1	400	.75	.61		1.067
Group E - Nominal t_s = 0.064 in.							
E-1	0.0633	∞	∞	0.99	0.51	0.0395	0.824
E-2	.0639	97.0	1518	1.00	.51	.0397	.824
E-3	.0639	55.7	872	.99	.52	.0400	.824
E-4	.0640	48.1	752	.99	.52	.0399	.822
E-5	.0640	43.1	673	.98	.51	.0399	.822
E-6	.0642	40.2	626	.98	.51	.0395	.830
E-7	.0640	35.0	547	.75	.50	.0395	.788
E-8	.0662	29.7	449	.76	.50	.0400	.799
E-9	.0645	25.7	398	.75	.51	.0402	.782
E-10	.0646	24.2	374	.86	.62	.0640	.942
E-11	.0644	16.7	259	1.00	.51	.0402	.779
E-12	.0647	14.7	227	.86	.64	.0645	.947
E-13	.0646	11.4	176	.86	.64	.0648	.944

[a]Group D consists of panels used in the tests reported in reference 2.
[b]Nominal dimensions.

TABLE II. - TEST RESULTS

(Data for specimens in group D taken from reference 2.)

Specimen	r/t	Buckling load (lb)	Buckling stress, σ_{cr} (lb/sq in.)	σ_{cr}/E	σ_{cr}/E for repeated loads			σ_{cr}/E (Specimen reground and retested)	σ_{cr}/E for second load after regrinding	Method of determining critical load
					Second	Third	Fourth			
A-1	∞	22,630	12,780	0.001206	--------	--------	--------	--------	--------	(a)
		19,400	10,950	.001033	--------	--------	--------	--------	--------	(b)
A-2	∞	21,190	12,160	.001147	--------	--------	--------	--------	--------	(a)
		18,000	10,330	.000975	--------	--------	--------	--------	--------	(b)
A-3	839	23,250	12,930	.001220	--------	--------	--------	--------	--------	(a)
		20,400	11,350	.001071	--------	--------	--------	--------	--------	(b)
A-4	776	23,020	12,950	.001222	--------	--------	--------	--------	--------	(a)
		19,700	11,080	.001045	--------	--------	--------	--------	--------	(b)
A-5	719	23,720	13,360	.001260	--------	--------	--------	--------	--------	(a)
		19,600	11,040	.001042	--------	--------	--------	--------	--------	(b)
A-6	694	23,830	13,290	.001254	--------	--------	--------	--------	--------	(a)
		19,500	10,880	.001026	--------	--------	--------	--------	--------	(b)
A-7	614	21,640	12,300	.001160	--------	--------	--------	--------	--------	(a)
		17,000	9,660	.000911	--------	--------	--------	--------	--------	(b)
A-8	524	24,740	13,840	.001306	--------	--------	--------	--------	--------	(a)
		22,000	12,300	.001160	--------	--------	--------	--------	--------	(b)
A-9	450	25,500	14,340	.001353	--------	--------	--------	--------	--------	(a)
		23,400	13,160	.001242	--------	--------	--------	--------	--------	(b)
A-10	356	27,220	15,110	.001425	0.001370	0.001370	0.001372	--------	--------	(c)
A-11	268	29,250	16,380	.001545	.001376	--------	--------	--------	--------	(c)
A-12	218	38,300	21,230	.002003	.001569	.001203	--------	--------	--------	(c)
A-13	132	54,700	30,590	.002886	--------	--------	--------	--------	--------	(c)
B-1	∞	14,220	9,500	.000896	--------	--------	--------	--------	--------	(a)
		13,250	8,850	.000835	--------	--------	--------	--------	--------	(b)
B-2	∞	13,560	9,130	.000861	--------	--------	--------	--------	--------	(a)
		11,800	7,950	.000750	--------	--------	--------	--------	--------	(b)
B-3	1227	13,730	9,320	.000879	--------	--------	--------	--------	--------	(a)
		12,100	8,210	.000775	--------	--------	--------	--------	--------	(b)
B-4	957	13,200	8,840	.000834	--------	--------	--------	--------	--------	(a)
		11,700	7,840	.000740	--------	--------	--------	--------	--------	(b)
B-5	935	14,300	9,450	.000892	--------	--------	--------	--------	--------	(a)
		13,000	8,590	.000810	--------	--------	--------	--------	--------	(b)
B-6	922	13,200	8,900	.000840	--------	--------	--------	--------	--------	(a)
		10,700	7,220	.000681	--------	--------	--------	--------	--------	(b)
B-7	760	14,310	9,670	.000912	--------	--------	--------	--------	--------	(a)
		11,200	7,570	.000714	--------	--------	--------	--------	--------	(b)
B-8	732	15,100	10,140	.000957	--------	--------	--------	--------	--------	(a)
		11,100	7,450	.000703	--------	--------	--------	--------	--------	(b)
B-9	729	14,490	9,700	.000915	--------	--------	--------	--------	--------	(a)
		11,500	7,700	.000726	--------	--------	--------	--------	--------	(b)
B-10	630	14,410	9,640	.000909	--------	--------	--------	--------	--------	(a)
		13,600	9,100	.000858	--------	--------	--------	--------	--------	(b)
B-11	551	14,260	9,700	.000915	--------	--------	--------	--------	--------	(a)
		13,500	9,180	.000866	--------	--------	--------	--------	--------	(b)
B-12	370	18,460	12,420	.001172	.001105	.001113	.001114	--------	--------	(c)
B-13	258	21,900	14,700	.001387	.001173	--------	--------	--------	--------	(c)
B-14	163	41,950	28,620	.002700	--------	--------	--------	--------	--------	(c)
C-1	∞	7,010	5,730	.000541	--------	--------	--------	0.000565	0.000577	(a)
		6,400	5,230	.000493	--------	--------	--------	.000478	.000486	(b)
C-2	1206	7,080	5,790	.000546	--------	--------	--------	.000540	--------	(a)
		6,600	5,400	.000509	--------	--------	--------	.000455	--------	(b)
C-3	737	7,300	5,980	.000564	--------	--------	--------	--------	--------	(b)
C-4	568	12,880	10,490	.000990	--------	--------	--------	.000677	--------	(c)
C-5	513	12,100	9,890	.000933	--------	--------	--------	.000628	.000626	(c)
C-6	427	11,050	9,100	.000858	--------	--------	--------	.000704	.000703	(c)
C-7	402	14,220	12,420	.001172	--------	--------	--------	.000611	.000611	(c)
C-8	348	16,500	14,510	.001369	--------	--------	--------	.000842	.000645	(c)
C-9	300	18,200	13,820	.001304	.001157	--------	--------	--------	--------	(c)
C-10	273	18,850	16,510	.001558	.001008	--------	--------	.001259	.000772	(c)
C-11	212	27,900	24,520	.002313	--------	--------	--------	.001041	.001024	(c)
C-12	188	32,750	25,370	.002393	.000541	.000537	--------	--------	--------	(c)
C-13	143	43,650	33,350	.003146	--------	--------	--------	--------	--------	(c)
D-1	∞	7,430	5,670	.000535	.000549	.000552	--------	--------	--------	(a)
D-2	1318	7,670	5,850	.000552	--------	--------	--------	--------	--------	(a)
D-3	806	6,250	5,400	.000509	--------	--------	--------	--------	--------	(b)
D-4	634	7,800	6,750	.000637	.000646	--------	--------	--------	--------	(b)
D-5	542	7,280	6,600	.000623	--------	--------	--------	--------	--------	(b)
D-6	478	9,030	8,190	.000773	--------	--------	--------	--------	--------	(b)
D-7	432	9,380	8,790	.000829	--------	--------	--------	--------	--------	(b)
D-8	400	11,050	10,360	.000977	--------	--------	--------	--------	--------	(c)
E-1	∞	3,460	4,200	.000396	--------	--------	--------	--------	--------	(a)
		2,360	2,860	.000270	--------	--------	--------	--------	--------	(b)
E-2	1518	3,200	3,880	.000366	--------	--------	--------	--------	--------	(b)
E-3	872	3,490	4,240	.000400	--------	--------	--------	.000404	--------	(c)
[d]E-4	752	------	------	--------	--------	--------	--------	.000335	--------	(b)
E-5	673	8,060	9,810	.000925	--------	--------	--------	--------	--------	(c)
E-6	626	3,310	3,990	.000376	.000382	--------	--------	.000372	.000369	(c)
E-7	547	3,100	3,930	.000371	.000375	--------	--------	.000540	--------	(c)
E-8	449	11,640	14,570	.001375	--------	--------	--------	.000574	.000573	(c)
E-9	398	6,600	8,440	.000796	.000790	--------	--------	.000539	.000539	(c)
E-10	374	6,960	7,390	.000697	.000679	--------	--------	--------	--------	(c)
E-11	259	14,700	18,870	.001780	--------	--------	--------	--------	--------	(c)
E-12	227	17,310	18,280	.001725	.000806	.000806	.000792	--------	--------	(c)
E-13	176	26,050	27,600	.002604	--------	--------	--------	--------	--------	(c)

[a]Straight-line method.
[b]Visual estimate from curve.
[c]Load at which snap-diaphragm action occurred.
[d]Load increments too large for determination of first buckling load.

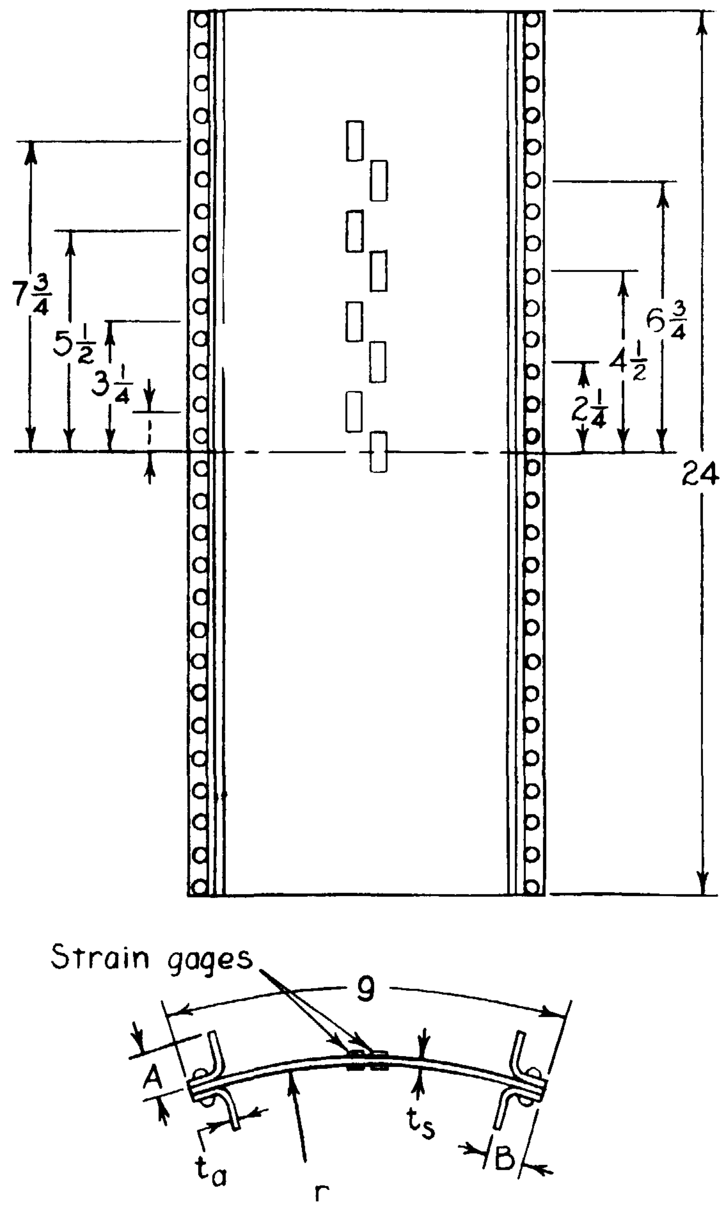


Figure 1.- Test specimen.

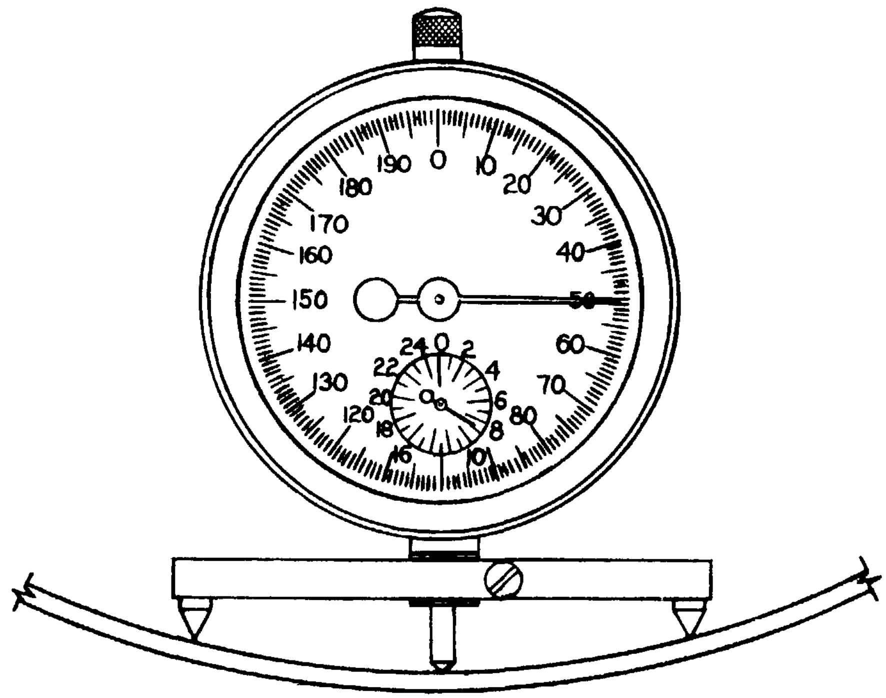


Figure 2. - Gage for measuring curvature

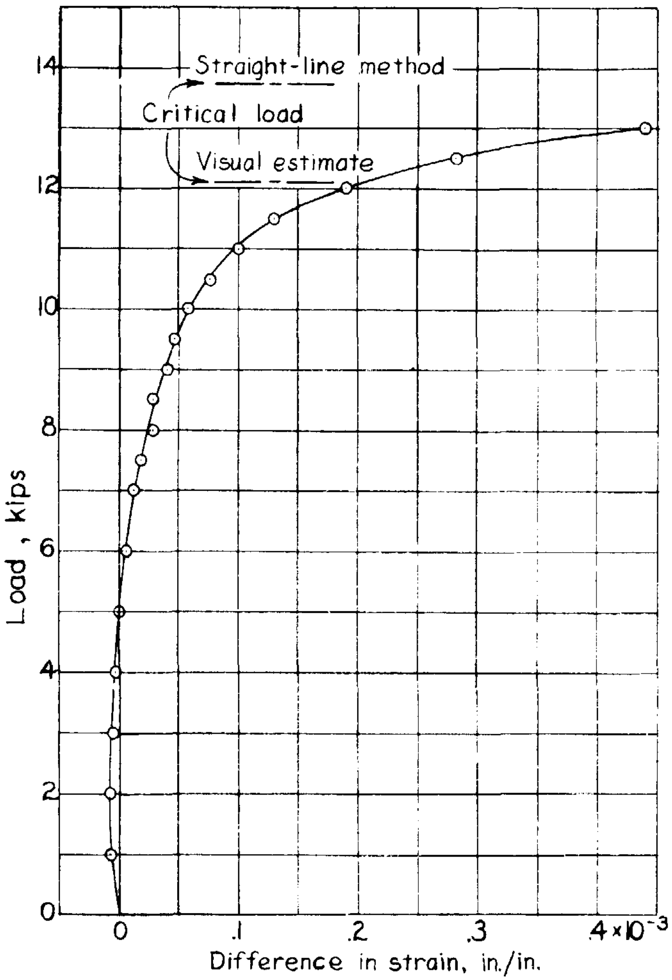
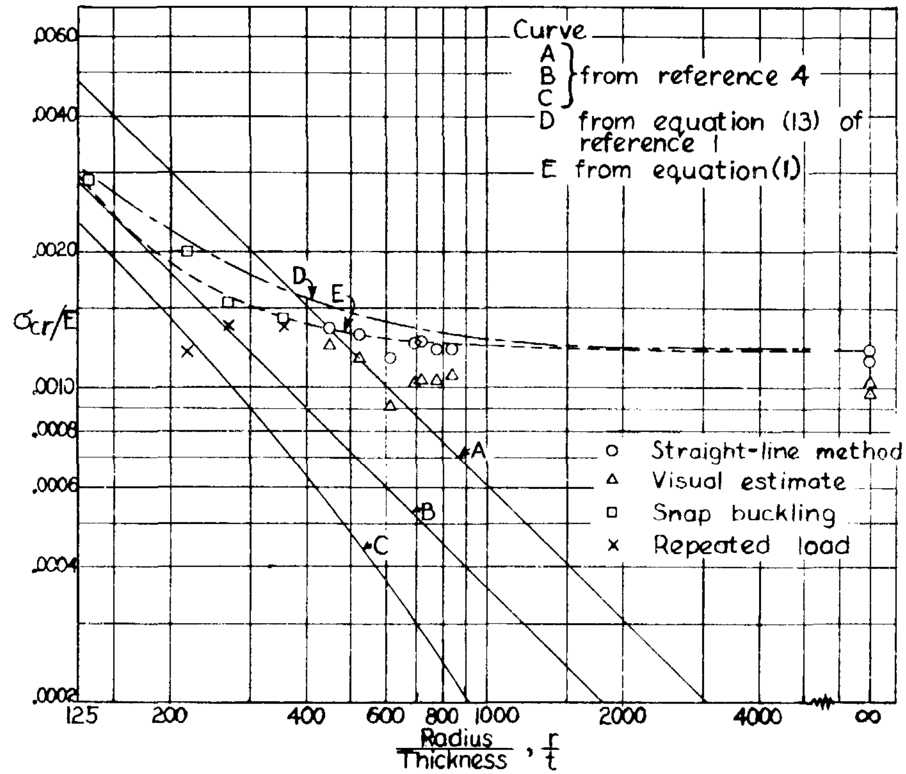
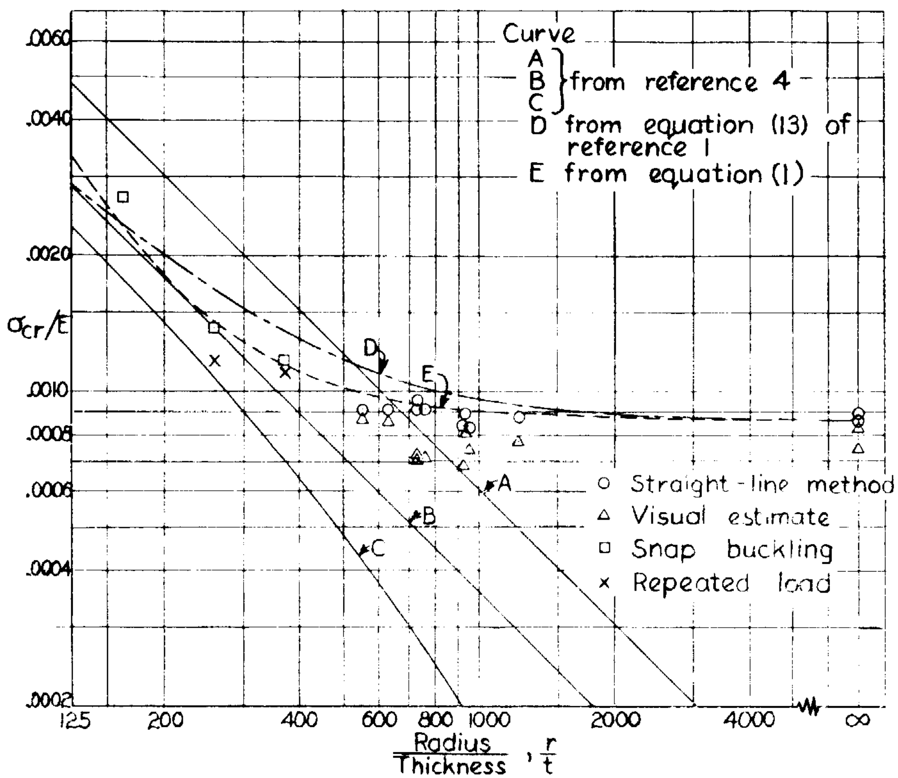


Figure 3.- Plot of difference in strain on opposite sides of sheet for visual estimate of critical load.



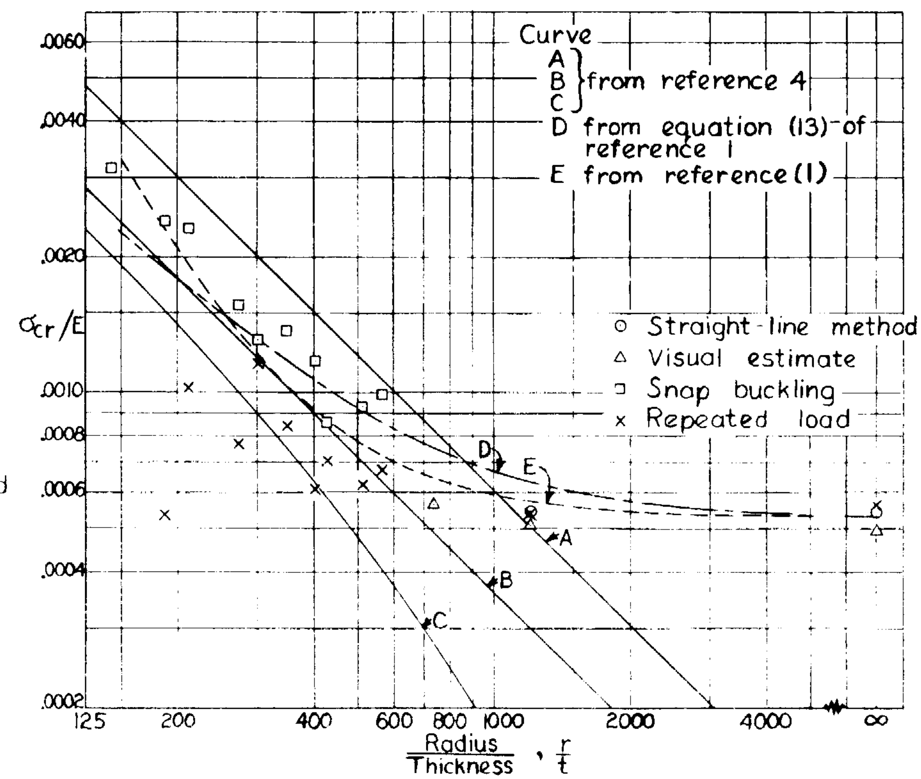
(a) Specimens, group A ; t_s = 0.125 in.

Figure 4.- Critical compressive stress for 24S-T aluminum-alloy curved sheet between stiffeners.



(b) Specimens, group B ; $t_s = 0.102$ in.

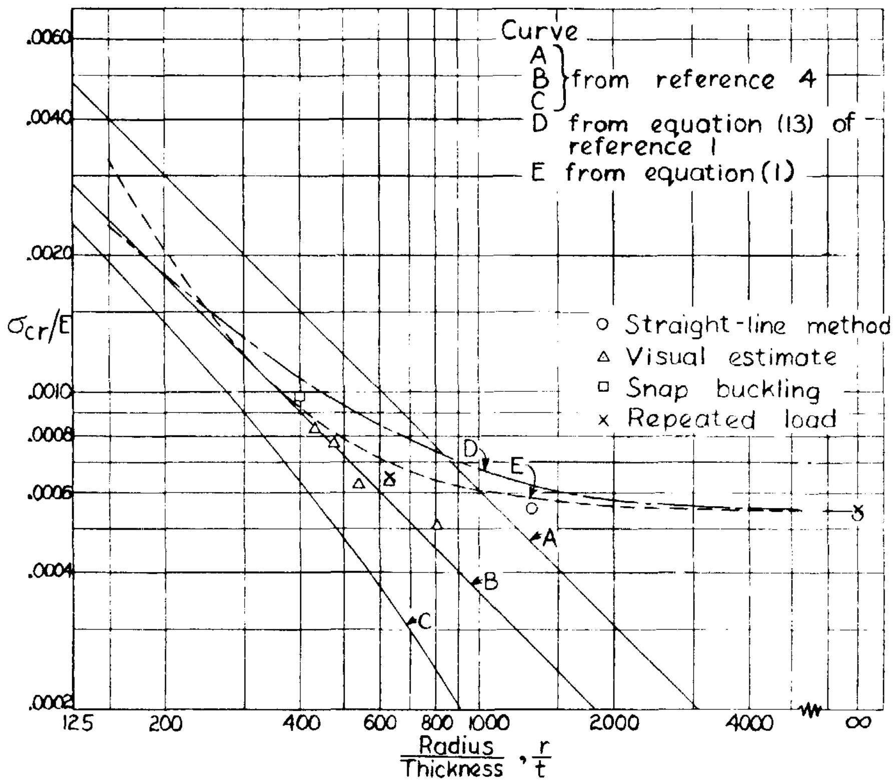
Figure 4. ~ Continued.



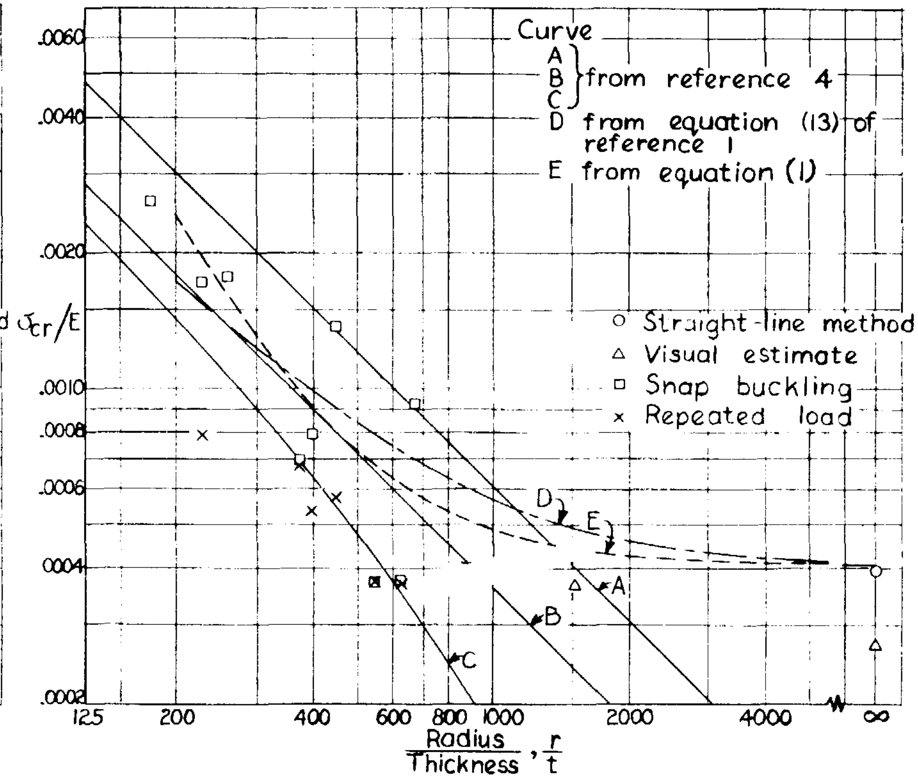
(c) Specimens, group C ; $t_s = 0.081$ in.

Figure 4 ~ Continued.

NACA



(d) Specimens, group D; t_s = 0.078 in.
Figure 4. ~ Continued.



(e) Specimens, group E; t_s = 0.064 in.
Figure 4. ~ Concluded.

Figs. 4d,e